OCR
Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

6386426023

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** **(i)** Given that $\mathrm{f}(x) = \arctan x$, write down an expression for $\mathrm{f}'(x)$. Assuming that x is small, use a binomial expansion to express $\mathrm{f}'(x)$ in ascending powers of x as far as the term in x^4. **[3]**

(ii) Hence express $\arctan x$ in ascending powers of x as far as the term in x^5. **[3]**

(b) Find, in exact form, the value of the following integral.

$$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,\mathrm{d}x$$

[5]

(c) A curve has polar equation $r = \dfrac{a}{\sqrt{\theta}}$ where $a > 0$.

(i) Sketch the curve for $\dfrac{\pi}{4} \leqslant \theta \leqslant 2\pi$. **[2]**

(ii) State what happens to r as θ tends to zero. **[1]**

(iii) Find the area of the region enclosed by the part of the curve sketched in part (i) and the lines $\theta = \dfrac{\pi}{4}$ and $\theta = 2\pi$. Give your answer in an exact simplified form. **[4]**

2 **(a)** **(i)** Express $2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta - \mathrm{j}\cos\frac{1}{2}\theta\right)$ in terms of z where $z = \cos\theta + \mathrm{j}\sin\theta$. **[3]**

(ii) The series C and S are defined as follows.

$$C = 1 - \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta - \ldots + (-1)^n\binom{n}{n}\cos n\theta$$

$$S = -\binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta - \ldots + (-1)^n\binom{n}{n}\sin n\theta$$

Show that

$$C + \mathrm{j}S = \left\{-2\mathrm{j}\sin\tfrac{1}{2}\theta\left(\cos\tfrac{1}{2}\theta + \mathrm{j}\sin\tfrac{1}{2}\theta\right)\right\}^n.$$

Hence show that, for even values of n,

$$\frac{C}{S} = \cot\left(\tfrac{1}{2}n\theta\right).$$

[8]

(b) Write the complex number $z = \sqrt{6} + \mathrm{j}\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, expressing r and θ as simply as possible.

Hence find the cube roots of z in the form $r\mathrm{e}^{\mathrm{j}\theta}$.

Show the points representing z and its cube roots on an Argand diagram. **[7]**

3 **(i)** Find the eigenvalues and eigenvectors of the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express **M** in the form $\mathbf{PDP}^{-1}$ where **D** is a diagonal matrix. **[8]**

(ii) Write down an equation for $\mathbf{M}^n$ in terms of the matrices **P** and **D**.

Hence obtain expressions for the elements of $\mathbf{M}^n$.

Show that $\mathbf{M}^n$ tends to a limit as n tends to infinity. Find that limit. **[6]**

(iii) Express $\mathbf{M}^{-1}$ in terms of the matrices **P** and **D**. Hence determine whether or not $(\mathbf{M}^{-1})^n$ tends to a limit as n tends to infinity. **[4]**

Section B (18 marks)

4 **(i)** Given that $y = \cosh x$, use the definition of $\cosh x$ in terms of exponential functions to prove that

$$x = \pm \ln\left(y + \sqrt{y^2 - 1}\right).$$

[5]

(ii) Solve the equation

$$\cosh x + \cosh 2x = 5,$$

giving the roots in an exact logarithmic form. **[5]**

(iii) Sketch the curve with equation $y = \cosh x + \cosh 2x$. Show on your sketch the line $y = 5$.

Find the area of the finite region bounded by the curve and the line $y = 5$. Give your answer in an exact form that does not involve hyperbolic functions. **[8]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4756/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

Section A (54 Marks)

1 (a) (i)	
1 (a) (ii)	

1 (b)	

1 (c) (i)	
1 (c) (ii)	
1 (c) (iii)	

2 (a) (i)

2 (a) (ii)	

2 (b)	

3 (i)	

3 (ii)	

3 (iii)	

Section B (18 Marks)

4 (i)

4 (ii)

4 (iii)	
	(answer space continued on next page)

4 (iii)	**(continued)**

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margins.

OCR
Oxford Cambridge and RSA

OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4756**: Further Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2016

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

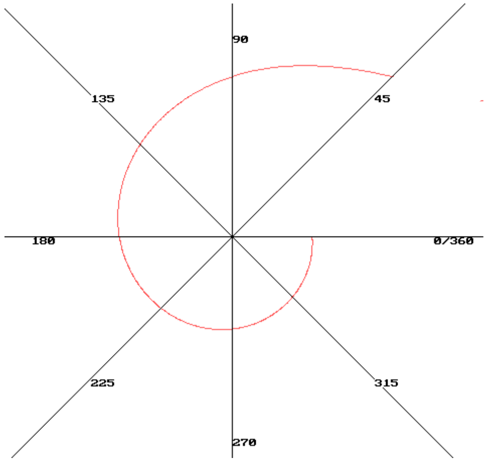
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

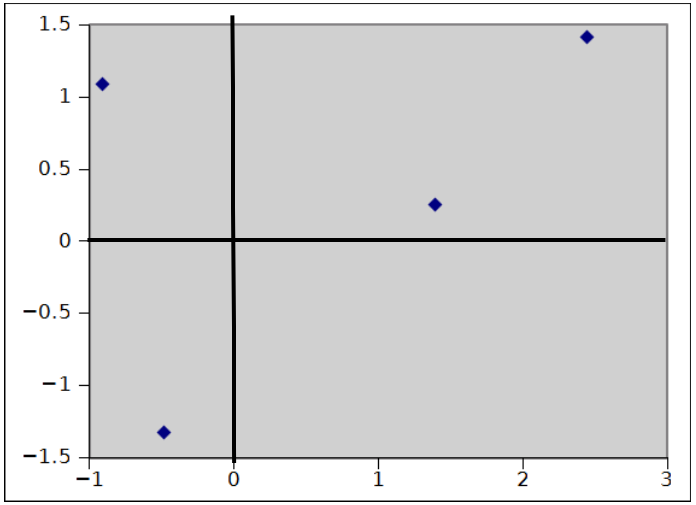
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(a)	(i)	$f'(x) = \frac{1}{1+x^2}$	B1		
			Binomial expansion gives	M1	Three terms from $(1+x^2)^{-1}$	
			$f'(x) = 1 - x^2 + x^4 (- \cdots)$	A1	*Give full marks for correct series*	*Ignore higher powers*
				[3]		
		(ii)	Integrate to obtain	M1		*Must use* $f'(x)$
			$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots (+c)$	A1		*Just answer (without +c) is M0*
			Use arctan(0) = 0 to find c = 0	A1		*Can be earned after M1A0*
				[3]		
	(b)		$\frac{1}{2}\int_0^{3/4} \frac{1}{\sqrt{\frac{3}{4} - x^2}} dx$	M1	For arcsin (or arccos)	or any sine (or cosine) substitution
			$\frac{1}{2}\left[\arcsin\frac{2x}{\sqrt{3}}\right]_0^{3/4}$	A1	For $\arcsin\frac{2x}{\sqrt{3}}$ (o.e.)	or $2x = \sqrt{3}\sin u$
				A1	For $\frac{1}{2}$	or $\left[\ \frac{1}{2}u\ \right]$
			$\frac{1}{2}\left(\arcsin\frac{\sqrt{3}}{2} - \arcsin 0\right)$	A1	For $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ soi	e.g. new limit is $\frac{\pi}{3}$
			$\frac{\pi}{6}$	A1		
				[5]		

Question			Answer	Marks	Guidance	
	(c)	(i)		G1 G1 [2]	Overall spiral shape *(lenient)* Correct limits for θ	
		(ii)	r tends to infinity as θ tends to zero	B1 [1]		
		(iii)	Area is $\frac{1}{2}\int_{\pi/4}^{2\pi}\frac{a^2}{\theta}\,\mathrm{d}\theta$ $\frac{1}{2}[a^2\ln\theta]_{\pi/4}^{2\pi}$ $\frac{1}{2}\left(a^2\ln 2\pi - a^2\ln\frac{\pi}{4}\right)$ Simplify to $\frac{3}{2}a^2\ln 2$	M1 A1 A1 A1 [4] [18]	For integral of $\left(\frac{a}{\sqrt{\theta}}\right)^2$ For $\ln 2\pi - \ln\frac{\pi}{4}$ o.e. Or $\frac{1}{2}a^2\ln 8$ or $a^2\ln(\sqrt{8})$ etc	

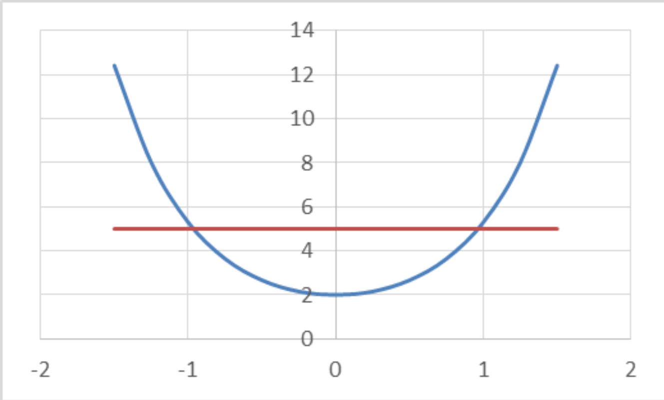
Question			Answer	Marks	Guidance	
2	(a)	(i)	$2\sin^2\left(\frac{1}{2}\theta\right) - \text{j}\,2\sin\left(\frac{1}{2}\theta\right)\cos\left(\frac{1}{2}\theta\right)$ $= (1-\cos\theta) - \text{j}\sin\theta$ $= 1 - z$	M1 A1 A1 **[3]**	Using half-angle formulae to express in terms of $\cos\theta,\ \sin\theta$	
		OR	$\left(\dfrac{z^{\frac{1}{2}} - z^{-\frac{1}{2}}}{\text{j}}\right)\left(\dfrac{z^{\frac{1}{2}} - z^{-\frac{1}{2}}}{2\text{j}} - \text{j}\,\dfrac{z^{\frac{1}{2}} + z^{-\frac{1}{2}}}{2}\right)$ $= \left(z^{\frac{1}{2}} - z^{-\frac{1}{2}}\right)\left(-\dfrac{z^{\frac{1}{2}} - z^{-\frac{1}{2}}}{2} - \dfrac{z^{\frac{1}{2}} + z^{-\frac{1}{2}}}{2}\right)$ $= 1 - z$		M1 A1 Correct form without j A1	
		OR	$-2\text{j}\sin\tfrac{1}{2}\theta\left(\cos\tfrac{1}{2}\theta + \text{j}\sin\tfrac{1}{2}\theta\right) = -(z^{\frac{1}{2}} - z^{-\frac{1}{2}})z^{\frac{1}{2}}$ $= 1 - z$		M1A1 A1	
		(ii)	$C + \text{j}\,S = 1 - \binom{n}{1}z + \binom{n}{2}z^2 - \ldots$ $= (1-z)^n$ Hence $C + \text{j}S = \left\{2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta - \text{j}\cos\frac{1}{2}\theta\right)\right\}^n$ $= \left\{(-\text{j})2\sin\frac{1}{2}\theta\left(\cos\frac{1}{2}\theta + \text{j}\sin\frac{1}{2}\theta\right)\right\}^n$ So $C + \text{j}S = (-2\text{j})^n\left(\sin\tfrac{1}{2}\theta\right)^n\left(\cos\tfrac{1}{2}n\theta + \text{j}\sin\tfrac{1}{2}n\theta\right)$ For n even, $\text{j}^n\ [= (-1)^{\frac{n}{2}} = \pm 1]$ is real	M1 A1 E1 M1 B1	 Applying deMoivre	 *May be implied*

Question			Answer	Marks	Guidance	
			Hence for n even, $C = (-2\text{j})^n \left(\sin\frac{1}{2}\theta\right)^n \cos(\frac{1}{2}n\theta)$	A1	For either C or S correct	*A0 for* $C = \cos(\frac{1}{2}n\theta)$ *and* $S = \sin(\frac{1}{2}n\theta)$
			and $S = (-2\text{j})^n \left(\sin\frac{1}{2}\theta\right)^n \sin(\frac{1}{2}n\theta)$	A1 ft	C with $\sin(\frac{1}{2}n\theta)$ for $\cos(\frac{1}{2}n\theta)$	
			So $\frac{C}{S} = \cot\left(\frac{1}{2}n\theta\right)$	E1 **[8]**	*Dependent on previous 4 marks*	
	(b)		Modulus $r = \sqrt{(6 + 2)} = 2\sqrt{2}$ (accept $\sqrt{8}$)	B1		
			Argument $\theta = \arctan(\sqrt{2} / \sqrt{6}) = \arctan(1 / \sqrt{3}) = \pi / 6$	B1		
			Cube roots: $r = \sqrt{2}$ (or $2^{\frac{1}{2}}$)	B1	B0 for $8^{1/6}$, 1.41	
			Arguments $\theta = \pi / 18, 13\pi / 18, 25\pi / 18$ (or $-11\pi / 18$)	B1B1	B1 for $\pi / 18$, B1 ft for other two	*Penalise missing* π*'s and* j*'s*
				G1 G1 **[7]**	For z For the cube roots	*Ignore scales and any distances marked*
				[18]		

Question		Answer	Marks	Guidance
3	(i)	$\det\begin{pmatrix} \frac{1}{2}-\lambda & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3}-\lambda \end{pmatrix} = 0$		
		$\left(\frac{1}{2}-\lambda\right)\left(\frac{1}{3}-\lambda\right) - \frac{1}{3} = 0$	B1	$6\lambda^2 - 5\lambda - 1 = 0$
		Roots $\lambda = 1, -1/6$	B1	
			M1	Using $\mathbf{Mx} = \lambda\mathbf{x}$ or $(\mathbf{M} - \lambda\mathbf{I})\mathbf{x} = 0$
		$\lambda = 1$: obtain $y = x$ hence eigenvector (e.g.) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	A1	
		$\lambda = -1/6$: obtain $3y = -4x$ hence eigenvector (e.g.) $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$	A1	
		$\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{6} \end{pmatrix}$	B1 ft	*For B1B1 the order must be consistent*
		$\mathbf{P} = \begin{pmatrix} 1 & 3 \\ 1 & -4 \end{pmatrix}$	B1 ft	
		$\mathbf{P}^{-1} = -\frac{1}{7}\begin{pmatrix} -4 & -3 \\ -1 & 1 \end{pmatrix}$	B1 ft **[8]**	The mark for $\mathbf{P}^{-1}$ may be gained in part (ii)

Question		Answer	Marks	Guidance	
	(ii)	$\mathbf{M}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$	B1	*Allow matrices written out provided*	*intention is clear*
		$\mathbf{D}^n = \begin{pmatrix} 1 & 0 \\ 0 & \left(-\frac{1}{6}\right)^n \end{pmatrix}$	B1 ft	$-\frac{1}{6}^n$ gets B0 unless recovered later	
		Multiply out $\mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$ to obtain	M1		
		$\frac{1}{7}\begin{pmatrix} 4+3\left(-\frac{1}{6}\right)^n & 3-3\left(-\frac{1}{6}\right)^n \\ 4-4\left(-\frac{1}{6}\right)^n & 3+4\left(-\frac{1}{6}\right)^n \end{pmatrix}$	A1	All terms required	A0 if not simplified e.g. 1^n
		As n tends to infinity, $\left(-\frac{1}{6}\right)^n$ tends to zero.	M1	*May be implied*	
		$\frac{1}{7}\begin{pmatrix} 4 & 3 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & \frac{3}{7} \\ \frac{4}{7} & \frac{3}{7} \end{pmatrix}$	A1 ft **[6]**		
	(iii)	$\mathbf{M}^{-1} = \mathbf{P}\mathbf{D}^{-1}\mathbf{P}^{-1}$	B1	*Allow matrices written out provided*	*intention is clear*
		$(\mathbf{M}^{-1})^n = \mathbf{P}\mathbf{D}^{-n}\mathbf{P}^{-1}$	M1	Or elements of $\left(\mathbf{M}^{-1}\right)^n$ are the same 'size' as elements of $\mathbf{D}^{-n}$	or M2 for $(\mathbf{M}^{-1})^n$ is the matrix in (ii) with n replaced by $-n$
		$\mathbf{D}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -6 \end{pmatrix}$ so $\mathbf{D}^{-n} = \begin{pmatrix} 1 & 0 \\ 0 & (-6)^n \end{pmatrix}$	M1	Or $\mathbf{D}^{-n}$ contains element $(-6)^n$	
		Hence $\left(\mathbf{M}^{-1}\right)^n$ does not tend to a limit	A1 **[4]**	*Dependent on M1M1*	

Question			Answer	Marks	Guidance	
4	(i)		$y = \frac{1}{2}(e^x + e^{-x})$	B1		
			Write as $t^2 - 2yt + 1 = 0$ where $t = e^x$	M1		
			Roots $t = e^x = y \pm \sqrt{(y^2 - 1)}$ o.e.	A1		
			Hence $x = \ln(y \pm \sqrt{(y^2 - 1)})$	A1		
			Show the roots are reciprocals of one another	E1	Answer given	
			So $x = \pm \ln(y + \sqrt{(y^2 - 1)})$			
				[5]		
	(ii)		$c + 2c^2 - 1 = 5$, where $c = \cosh(x)$	B1		
			Solve quadratic	M1		
			$c = 3/2$	A1		
			Other root ($c = -2$) rejected	A1		
			Obtain $x = \pm \ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$	A1	or $\ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$	
				[5]		
		OR			M1 Quartic in e^x, factorised	
			$(e^{2x} - 3e^x + 1)(e^{2x} + 4e^x + 1) = 0$		A1	
			$e^x = \frac{3 \pm \sqrt{5}}{2}$		A1	
			Other roots ($e^x = -2 \pm \sqrt{3}$) rejected		A1	
			$x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$		A1	

Question			Answer	Marks	Guidance	
	(iii)			G2	Fully correct, including $(0, 2)$ Give G1 for U-shaped curve symmetrical about the *y*-axis	
			Area beneath the curve: $\int_{-a}^{a}(\cosh x + \cosh 2x)\,dx$ where $a = \ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$			
			$\left[\sinh x + \frac{1}{2}\sinh 2x\right]_{-a}^{a}$	B1B1	For $\sinh x$ and $\frac{1}{2}\sinh 2x$	Might be in exponential form
			$[\sinh x(1 + \cosh x)]_{-a}^{a}$ $2\sinh a(1 + \cosh a)$	M1	Substituting limit $x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$	
			$2\sqrt{\frac{5}{4}}\left(1 + \frac{3}{2}\right)$	A1	For $\cosh a = \frac{3}{2}$ and $\sinh a = \frac{\sqrt{5}}{2}$	or $e^a = \frac{3+\sqrt{5}}{2}$ and $e^{2a} = \left(\frac{3+\sqrt{5}}{2}\right)^2$
			$\frac{5\sqrt{5}}{2}$	A1		
			Required area is $10\ln\left(\frac{3+\sqrt{5}}{2}\right) - \frac{5\sqrt{5}}{2}$	B1 ft **[8]**	For 10×(answer to (ii)) – (area under curve)	
				[18]		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

4756 Further Methods for Advanced Mathematics (FP2)

General Comments:

Most candidates appeared to have sufficient time to complete the paper, and were able to demonstrate a sound understanding of the topics being examined. Q.1 (on inverse circular functions and polar coordinates) was the best answered question, and Q.2 (on complex numbers) was the worst answered.

Comments on Individual Questions:

Q.1(a)(i) Almost all candidates wrote down f '(x) and obtained the binomial series correctly. The only common errors were incorrect signs.

Q.1(a)(ii) The series was usually obtained by integrating the series from part (i), but most candidates did not score full marks on this part. Very many candidates did not mention the constant of integration at all, and many that did left + c in their answer, omitting to show that the constant was zero.

Q.1(b) This integration was very well done. Errors such as arcsin($4x/3$) instead of arcsin($2x/\sqrt{3}$), and omitting the factor ½, were fairly common.

Q.1(c)(i) The curve was usually drawn well, although some continued the curve beyond the domain required.

Q.1(c)(ii) Most candidates correctly stated that r tends to infinity; although some just wrote 'r increases', which was not an adequate answer. Many candidates thought that r tends to zero.

Q.1(c)(iii) The enclosed area was usually obtained correctly.

Q.2(a)(i) Candidates who used the half-angle formulae quickly obtained $1 - \cos\theta - \text{j}\sin\theta$ and hence $1 - z$. However, many candidates chose to express everything in terms of $z^{½}$, and this approach was much less successful, with many sign errors and missing j's; for example, $z^{½} - z^{-½} = 2\sin½\theta$ was a common starting point.

Q.2(a)(ii) Most candidates scored 3 marks or fewer (out of 8) on this part. $C + \text{j}\,S = (1 - z)^n$ was commonly obtained, and this was quite often rearranged into the given form, using part (i) or otherwise. Candidates were then expected to give explicit expressions for C and S by taking real and imaginary parts. It was crucial to state that j^n is real when n is even, but most candidates did not do this. Also, $(\sin½\theta)^n$ often became $\sin½n\theta$.

Q.2(b) Most candidates understood the exponential form of a complex number, and knew how to obtain the cube roots. There were some careless slips such as omitting j or π from the exponent, and some candidates did not divide the argument by 3 when finding the cube roots. The modulus of the cube roots was sometimes left as $8^{1/6}$ or $(2\sqrt{2})^{1/3}$ without being simplified to $\sqrt{2}$. The cube roots were usually indicated on the Argand diagram correctly, but a very large number overlooked the request to show z on the diagram.

Q.3(i) The methods for finding eigenvalues and eigenvectors was very well understood, and most candidates scored full marks on this part.

Q.3(ii) Most candidates wrote down the correct expression $\mathbf{M}^n = \mathbf{PD}^n\mathbf{P}^{-1}$, but the element $(-1/6)^n$ in $\mathbf{D}^n$ very often became $-(1/6)^n$ leading to incorrect evaluation of the elements of $\mathbf{M}^n$. The limiting value as n tends to infinity was very often found correctly.

Q.3(iii) Many candidates wrote $\mathbf{M}^{-1} = \mathbf{P}^{-1}\mathbf{D}^{-1}\mathbf{P}$ instead of $\mathbf{M}^{-1} = \mathbf{P}\mathbf{D}^{-1}\mathbf{P}^{-1}$. Candidates were expected to argue, in a similar way to part (ii), that the elements of $(\mathbf{M}^{-1})^n$ contained $(-6)^n$ and so did not tend to a limit. However, the explanations were very often unclear, and most candidates scored no marks or 1 mark (out of 4) in this part.

Q.4(i) Most candidates showed that $x = \ln(y \pm \sqrt{(y^2 - 1)})$, but very many could not prove the final step $\ln(y - \sqrt{(y^2 - 1)}) = -\ln(y + \sqrt{(y^2 - 1)})$. The solution $x = \ln(y - \sqrt{(y^2 - 1)})$ was often rejected as being undefined.

Q.4(ii) Most candidates obtained a quadratic equation in $\cosh x$ and completed this successfully. Some wrote the equation in exponential form, but these rarely made much progress.

Q.4(iii) The curve was usually sketched correctly, although the y-intercept was often missing or incorrect. Finding the area under the curve in an exact simplified form caused many difficulties. The simplest way was to write the integrated expression as $\sinh x(1 + \cosh x)$ and then substitute $\sinh x = \pm\frac{1}{2}\sqrt{5}$ and $\cosh x = 3/2$; but most candidates changed it into exponential form, which made the substitution much more complicated. Many candidates omitted the final step of subtracting the area under the curve from the area of a rectangle to obtain the area of the specified region.

Oxford Cambridge and RSA

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	57	52	47	42	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	49	42	35	29	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	64	57	51	45	39	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	59	53	48	43	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	60	54	48	43	38	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	60	54	49	44	39	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	67	61	55	49	43	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	59	53	47	41	36	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	60	53	46	40	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	60	54	48	42	37	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	55	50	45	40	36	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	55	49	44	39	33	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0

	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	48	43	38	34	30	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

		Max Mark	a*	a	b	c	d	e	u
H860	01 Mathematics for Engineering	This unit has no entries in June 2016							
H860	02 Mathematics for Engineering								

Level 3 Certificate Mathematical Techniques and Applications for Engineers

			Max Mark	a*	a	b	c	d	e	u
H865	01 Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H866	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H866	02 Critical maths	Raw	60	47	41	35	29	23	0
		Overall	132	111	96	81	66	51	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H867	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H867	02 Statistical problem solving	Raw	60	40	34	28	23	18	0
		Overall	132	103	88	73	59	45	0

Advanced Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6993	01 Additional Mathematics	Raw	100	59	51	44	37	30	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

Oxford Cambridge and RSA

Version	Details of change
1.1	Correction to Overall grade boundaries for H866
	Correction to Overall grade boundaries for H867